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D7 brane embeddings and chiral symmetry breaking

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Abstract: We study the embedding of D7 brane probes in five geometries that are deformations of $AdS_5 \times S^5$. Each case corresponds to the inclusion of quark fields in a dual gauge theory where we are interested in investigating whether chiral symmetry breaking occurs. We use a supersymmetric geometry describing an $\mathcal{N}=2$ theory on its moduli space and a dilaton driven non-supersymmetric flow to establish criteria for a chiral symmetry breaking embedding. We develop a simple spherical D7 embedding that tests the repulsion of the core of the geometry and signals dynamical symmetry breaking. We then use this tool in more complicated geometries to show that an $\mathcal{N}=2^*$ theory and a non-supersymmetric theory with scalar masses do not induce a chiral condensate. Finally we provide evidence that the Yang Mills* geometry does.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

A pleasing geometric picture of chiral symmetry breaking in strongly coupled gauge theory has emerged from dual brane constructions [1]–[8]. Quark fields can be introduced into the AdS/CFT Correspondence [9]–[11] by the introduction of new branes [12]–[30]; strings stretched between these branes and the branes of the original construction transform in the fundamental representations of a product gauge group. The scalar field describing the separation of these two types of branes is dual to the quark mass and quark condensate operator $\bar{\psi}\psi$. The embedding of the new branes in the space created by the original therefore provides a dynamical determination of whether there is a fermion bilinear operator that breaks chiral symmetry. Such a symmetry is normally a manifest symmetry of the spacetime, and the brane's embedding can be explicitly seen to dynamically break it. These constructions allow the study of a number of phenomenological aspects of strongly coupled gauge theories including the pions and thermal properties [1]–[8].

So far this phenomena has been studied in a small number of geometrically simple spacetimes. The simplest cases consist of placing a probe D7 brane into a geometry that is a deformation of $AdS_5 \times S^5$. Asymptotically at large radius or high energy the theory becomes the $\mathcal{N} = 2$ theory with hypermultiplets of Karch and Katz [12], which is well understood. When the deformation is caused by the R-chargeless dilaton switching on [37]–[40], the SO(6) symmetry of the space is untouched and the embedding equations for the probe D7 are easily solved to give a profile dependent on the radial direction in the space [1]–[5]. Similarly, deformations due to the inclusion of finite temperature [1] leave the S^5 unspoilt and allow a simple analysis. Typically such deformations have a singular core to the geometry indicating the presence of a multi-centre like or fuzzy D3 brane distribution.

In non-supersymmetric configurations, which can allow the fermion condensate, these are hard to resolve, but in chiral symmetry breaking cases it is the repulsion of the probe D7 by this core that cause the fermion condensate. We can escape without a precise knowledge of the singularity.

In [2] a parallel analysis studies a six dimensional theory reduced by compactification to a four dimensional gauge theory in the infra-red (IR). It again has a repulsive, but this time non-singular, core that induces chiral symmetry breaking. Again the space-time is sufficiently simple that the embedding equations may be straightforwardly solved.

It would be nice to have a wider understanding of how generic the phenomena of chiral symmetry breaking is in gravitational duals - naively one might expect any non-supersymmetric, strongly coupled gauge background to induce chiral symmetry breaking. In this paper we pursue this goal by studying D7 brane embeddings in a wider range of deformations of $AdS_5 \times S^5$. In general such geometries are considerably more complicated and the embedding profile depends on many of the coordinates on the brane. This turns finding the solutions into an extended and complicated numerical problem. Instead we propose a simple spherical embedding of a D7 brane that can be performed analytically and tests the repulsion of the core of the geometry. We believe this is a good indication of whether chiral symmetry would be induced. Further, in the case of a known dilaton induced flow, it provides an analytic estimate of the dynamical quark mass at zero energy which matches the numerical results well.

We begin by looking at a deformed geometry of Freedman, Gubser, Pilch and Warner [31]. In fact, this geometry is a coordinate transformation of an $\mathcal{N} = 4$ preserving multi-centre solution in which chiral symmetry breaking can not occur. Study of it though highlights the pitfalls of interpreting the numerical embedding solutions too glibly. In one coordinate system we find solutions for the D7 brane embedding that appear to give a non-zero value for the quark condensate in the ultra-violet (UV) when in fact there is none. We learn that for there to be chiral symmetry breaking there must be a spatial gap between the embedded D7 brane and the core singularity for the case of a massless quark in the UV.

We next quickly review the dilaton induced flow results [1]–[5] and introduce our spherical D7 embedding showing how the full solutions corresponding to the usual D7 embedding match on to these solutions in the infra-red. The spherical embedding illustrates the repulsion of the core of the geometry and provides the analytic result for the induced mass gap in the theory.

As a first new non-supersymmetric geometry we look at an extension of the FGPW solution [41] which allows for the inclusion of non-zero masses for the adjoint scalars of the $\mathcal{N} = 4$ gauge theory as well as a vacuum expectation value. The gauge theory here has an unbounded scalar potential but the supersymmetry breaking could induce chiral symmetry breaking for the probe. We find that this is a case where chiral symmetry breaking is not present.

Pilch and Warner have constructed a number of geometries describing the $\mathcal{N} = 2^*$ gauge theory [42, 44, 45]. We study a D7 embedding that preserves the $\mathcal{N} = 2$ symmetry. The spherical D7 embedding allows us to check that there is no repulsion in this case and hence no chiral symmetry breaking as one expects in a supersymmetric theory.

Finally we look at the non-supersymmetric Yang Mills* geometry [46]. Here the deformation is instigated by a mass and or condensate for the adjoint fermions of the $\mathcal{N}=4$ theory (one would expect these parameters to be linked but the supergravity geometry provides inconclusive evidence for what this link is). This supersymmetry breaking induces a mass for the adjoint scalars too leaving just the Yang Mills field to survive to the IR. One would expect this theory to break chiral symmetries. We find using the spherical embedding that generically the core is repulsive (although there is a line of flows in the adjoint mass vs condensate plane where chiral symmetry breaking is absent).

2. D7 Branes and $\mathcal{N}=4$ Geometries

The large N_c $\mathcal{N}=4$ gauge theory at the origin of its moduli space is dual, via the AdS/CFT Correspondence to supergravity on $AdS_5 \times S^5$ with radius R

$$ds^2 = \frac{u^2}{R^2} dx_{//}^2 + \frac{R^2}{u^2} \sum_{i=1}^{6} du_i^2\,, \tag{2.1}$$

where $x_{//}$ is the 3+1d plane parallel to the D3 world volume and $u^2 = \sum_i u_i^2$. Quark fields may be introduced [12] and $\mathcal{N}=2$ supersymmetry preserved by placing a D7 probe in the $x_{//}$ and $u_1 - u_4$ directions (we write the metric on these four directions as $d\rho^2 + \rho^2 d\Omega_3^2$). The U(1) symmetry in the $u_5 - u_6$ plane is the geometric realization of the $\mathrm{U}(1)_A$ axial symmetry of the quark fields on the probe. The Dirac Born Infeld action for the probe, with tension T_7, is

$$S \;=\; -T_7 \int d^8\xi \sqrt{-\mathrm{det} G_{ab}} \;=\; -T_7 \int d^4x\; d\rho\; d\Omega_3 \rho^3 \sqrt{1 + u_5'^2 + u_6'^2}\,, \tag{2.2}$$

where the prime indicates a derivative with respect to ρ, G_{ab} is the pullback of the metric onto the probe, and we generically use $d^8\xi$ to indicate integration over the world volume. The resulting equation of motion for the profile in, for example, the u_6 direction is:

$$\frac{d}{d\rho}\left[\frac{\rho^3 u_6'}{\sqrt{1+u_6'^2}}\right] = 0\,. \tag{2.3}$$

The regular solution is $u_6 = m$ with the separation of the D7 from the ρ axis, m, representing the hypermultiplet's mass. Asymptotically at large u there is a second solution

$$u_6 = m + \frac{c}{\rho^2} + \cdots\,. \tag{2.4}$$

The parameter c represents the magnitude of the fermionic quark bilinear $\bar\psi\psi$. In pure AdS this solution is singular in the interior and unphysical [1].

Now as a first example of embedding a D7 brane in a deformed geometry we consider the geometry of Gubser, Freedman, Pilch and Warner [31]. This geometry was constructed in the 5d truncation of IIB supergravity on AdS [32]–[34] then lifted to 10d. A scalar field, χ, in the 20 dimensional representation of $\mathrm{SU}(4)_R$ has a non trivial radial profile corresponding to the scalar operator $\mathrm{Tr}\,\phi^2$ having a vev of the form diag$(1,1,1,1,-2,-2)$. This vev preserves the supersymmetry of the $\mathcal{N}=4/2$ theory and we would expect to be able to introduce quarks of any mass via a D7 brane and find no chiral symmetry breaking. This is indeed the case, as we will show, but there are some interesting subtleties.

The 10d geometry is given by:

$$ds^2 = \frac{X^{1/2}}{\chi} e^{2A} dx_{//}^2 - \frac{X^{1/2}}{\chi}\left(du^2 + \frac{R^2}{\chi^2}\left[d\theta^2 + \frac{\sin^2\theta}{X} d\phi^2 + \frac{\chi^6\cos^2\theta}{X} d\Omega_3^2\right]\right). \tag{2.5}$$

Here $d\Omega_3^2$ is the metric of a three sphere, R is the asymptotic (large u) AdS radius, and

$$X = \cos^2\theta + \chi^6 \sin^2\theta\,. \tag{2.6}$$

The fields χ and A satisfy the differential equations

$$\frac{d\chi}{du} = \frac{1}{3R}\left(\frac{1}{\chi} - \chi^5\right), \qquad \frac{dA}{du} = \frac{2}{3R}\left(\frac{1}{\chi^2} + \frac{\chi^4}{2}\right), \tag{2.7}$$

with solution

$$e^{2A} = \frac{l^2}{R^2}\frac{\chi^4}{\chi^6 - 1}, \tag{2.8}$$

where l is an integration constant. The solution also has a non-zero four-form potential

$$C_{(4)} = \frac{e^{4A} X}{g_s \chi^2} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\,. \tag{2.9}$$

Note that this solution is singular at the point where $\chi = 1$ - we will see the physical interpretation of this shortly.

It is natural to embed the D7 brane in this geometry to lie in the $x_{//}, u, \Omega_3$ directions and look at the profile of the D7 brane $\theta(u)$ at fixed ϕ (ϕ provides the U(1) symmetry of the embedding which is the chiral symmetry of the theory with quarks). The DBI action is

$$S_{DBI} = -T_7 \int d^8\xi\; e^{4A(u)} R^3 |\cos^3\theta(u)| \sqrt{X}\chi(u)^2 \sqrt{1 + \frac{\theta'(u)^2 R^2}{\chi(u)^2}}\,. \tag{2.10}$$

To place the solutions in a cartesian like plane instead we can make a change of coordinates from the circular coordinates (u,θ) into the set (r,v):

$$\begin{aligned} r^2 + v^2 &= e^{2u}\,, \\ \frac{v}{r} &= \tan\theta\,. \end{aligned} \tag{2.11}$$

Using this coordinate transformation, the metric becomes:

$$\begin{aligned} ds^2 = \frac{\sqrt{X}}{\chi} e^{2A} dx_{//}^2 - \frac{\sqrt{X}}{\chi\,(v^2+r^2)^2}\Bigg(& dv^2\left(v^2 + \frac{R^2 r^2}{\chi^2}\right) + dr^2\left(r^2 + \frac{R^2 v^2}{\chi^2}\right) + \\ & + 2rv\,dv\,dr\left(1 - \frac{1}{\chi^2}\right) + \frac{R^2\left(v^2+r^2\right)^2}{X} \times \\ & \times \left(\frac{1}{\chi^2\left(r^2/v^2+1\right)} d\phi^2 + \frac{\chi^4}{\left(1+v^2/r^2\right)} d\Omega_3^2\right)\Bigg). \end{aligned} \tag{2.12}$$

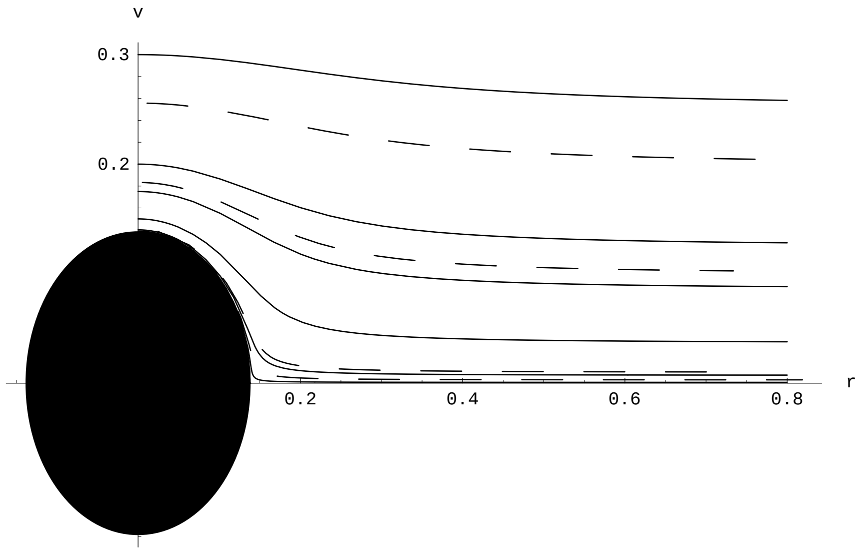

Figure 1: D7 brane flow in the FGPW geometry. The solid lines are the numerical solutions and the dashed lines the coordinate transform of the full analytic solutions. The singularity of the geometry is shown as a black circle.

Note that in these coordinates v and r are not perpendicular. In the AdS limit r, v become ρ, u_6 above though, and hence the embedding results will be easier to interpret in these coordinates where we can compare them to the AdS $u_6 = m$ result.

From the metric we can calculate the action when we embed with v and ϕ as the perpendicular directions. Again we choose $\phi = const$ and now our variable is $v(r)$.

$$S_{DBI} = -T_7 \int d^8\xi \, \frac{e^{4A} R^3 \chi^2 \sqrt{X}}{v^2 + r^2} \frac{1}{(1 + v^2/r^2)^{3/2}} \times \\ \times \sqrt{r^2 + \frac{R^2 v^2}{\chi^2} + 2rv\left(1 - \frac{R^2}{\chi^2}\right) v' + \left(v^2 + \frac{R^2 r^2}{\chi^2}\right) v'^2} \,,$$

where

$$\chi = \chi\left(\frac{1}{2}\log(v(r)^2 + r^2)\right), \qquad A = A\left(\frac{1}{2}\log(v(r)^2 + r^2)\right). \tag{2.13}$$

We can solve the equations of motion resulting from this action numerically and the solutions are shown in figure 1. Note that the D7 appears to deform around the singularity and there is a non-zero gradient at large r. Naively therefore our embedding solutions suggest that there is a quark condensate present for some values of the quark mass — we plot the value extracted asymptotically also in figure 2. If there were a quark condensate for any value of the mass supersymmetry would be broken which would be a surprise!

In fact, this result is an artifact of being in the wrong coordinates for the duality to the field theory to be manifest. In general finding the appropriate coordinates is rather hard but in this highly supersymmetric theory the correct coordinates have been identified

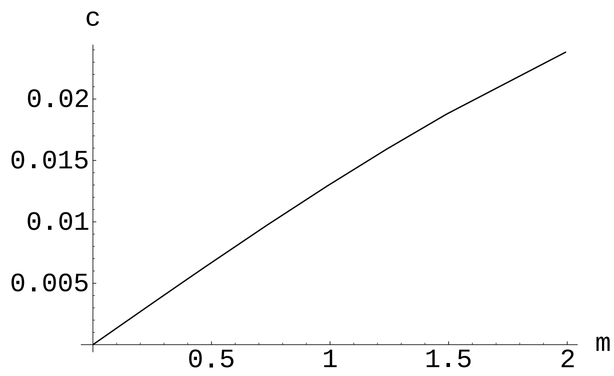

Figure 2: Apparent values of the quark mass and condensate extracted asymptotically from the flows in figure 1.

in [31, 36]. If we change the coordinates u, θ for $\hat{u}, \alpha$ so

$$\hat{u}^2 \cos^2 \alpha = R^2 e^{2A} \chi^2 \cos^2 \theta \,, \qquad \hat{u}^2 \sin^2 \alpha = R^2 \frac{e^{2A}}{\chi^4} \sin^2 \theta \,, \tag{2.14}$$

then the metric takes the form of a standard multi-centre D3 brane solution:

$$ds^2 = H^{-1/2} dx_{//}^2 + H^{1/2} \sum_{i=1}^{6} d\hat{u}_i^2 \,, \qquad C_{(4)} = \frac{1}{H g_s} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,, \tag{2.15}$$

with

$$H = \frac{\chi^2}{X e^{4A}} \,. \tag{2.16}$$

The singular region of the space is transformed from a sphere to a disc by this transformation. The disc lies at the origin in the v, r space. The singularity is now understood to be caused by the presence of a disc of D3 branes and the solution is a perfectly good geometry.

Embedding the D7 brane in the $x_{//}$ and four of the $\hat{w}_i$ directions leaves the standard DBI action (eq. (2.2)) for the D7 in AdS space - the factors of H cancel and play no part. There are as usual solutions with $\hat{u}_5, \hat{u}_6$ constant as in AdS. We interpret this as quark fields with a non-renormalized mass and no quark condensate.

Using the change of coordinates in eq. (2.14) we can map these simple $\hat{u}_6$= constant solutions onto the solutions shown in figure 1. These are given by the dashed lines showing that the numerical solutions to the equations of motion do match with those acquired from the analytic functions of χ and A. There are a number of lessons we can learn from this example. Firstly, it is not straightforward to interpret the D7 embedding solutions in terms of the field theory because of the potential ambiguity in the identification of the coordinate system. We can see though there were two signals of the absence of chiral symmetry breaking in this system even in the wrong coordinates. Firstly, the value of the parameter c in the solutions fell to zero as m fell to zero. Secondly, the embedding solutions filled the full space down to the singularity — that is there was no clear radial gap between the embedding solutions and the singularity. The result of this is that, as we have seen, the singularity can be transformed to a branch cut by a coordinate transformation and the solution for a massless quark then lies along the $\hat{u}_6 = 0$ axis. A true signal of chiral symmetry breaking, as we'll see shortly, is if there is a radial gap between the $m = 0$ embedding and the singularity — such an energy scale gap can never be removed by a coordinate transformation.

3. Quarks in a dilaton flow geometry

Chiral symmetry breaking was first observed with a D7 embedding [1] in the non-supersymmetric geometry of Constable and Myers [37]. This geometry in *Einstein frame* is given by

$$ds^2 = H^{-1/2}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta/4} dx_4^2 + H^{1/2}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{(2-\delta)/4} \frac{w^4-b^4}{w^4}\sum_{i=1}^{6} dw_i^2\,, \tag{3.1}$$

where

$$H = \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta} - 1\,, \tag{3.2}$$

and the dilaton and four-form are

$$e^{2\Phi} = e^{2\Phi_0}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{\Delta}, \qquad C_{(4)} = -\frac{1}{4}H^{-1} dt\wedge dx\wedge dy\wedge dz\,. \tag{3.3}$$

There are formally two free parameters, R and b, since

$$\delta = \frac{R^4}{2b^4}\,, \qquad \Delta^2 = 10 - \delta^2\,. \tag{3.4}$$

It is convenient to write w and b in units of R which removes R from the definition of δ and puts a factor of R^2 into g_{ww}. The parameter b has no R-charge and dimension four so is interpreted as the vev of the operator $\mathrm{Tr}\, F^2$. The $\mathcal{N} = 4$ gauge theory is not expected to have a vev for this operator and hence this geometry probably does not describe a physical gauge theory vacuum. Nevertheless, it is an interesting geometry to study chiral symmetry breaking in because it does describe a non-supersymmetric, strongly coupled gauge background.

This geometry is particularly simple to study because it has a flat six plane transverse to the D3s. We parameterize this six plane as

$$\sum_{i=1}^{6} dw_i^2 = d\rho^2 + \rho^2 d\Omega_3^2 + dw_5^2 + dw_6^2\,, \tag{3.5}$$

where we will embed the D7 brane on the directions $x_{//}$, ρ and Ω_3. The U(1) symmetry in the transverse $w_5 - w_6$ plane is the geometric realization of the $\mathrm{U}(1)_A$ symmetry of the quark fields. The DBI action for the D7 is

$$S = -T_7\int d^8\xi\, \rho^3\left(\frac{(\rho^2+w_6^2)^4 - b^8}{(\rho^2+w_6^2)^4}\right)\left(\frac{(\rho^2+w_6^2)^2+b^4}{(\rho^2+w_6^2)^2-b^4}\right)^{\frac{\Delta}{2}}\sqrt{1+(\partial_\rho w_6)^2}\,, \tag{3.6}$$

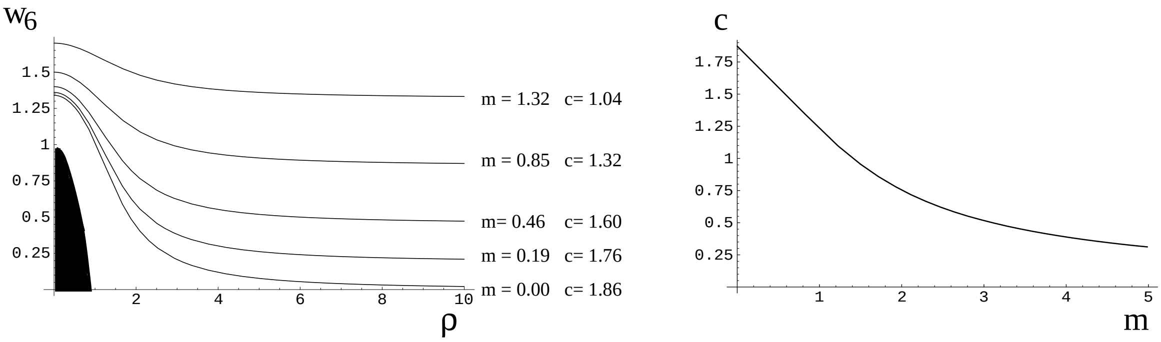

Figure 3: Numerical solutions for D7 embeddings in the Constable Myers geometry with $b = 1$. The shaded area corresponds to the singularity in the metric. Also plotted is the value of the quark condensate vs the mass extracted asymptotically from those flows.

and we look for solutions where $w_5 = 0$ and w_6 is a function of ρ. The equation of motion is

$$\frac{d}{d\rho}\left[\frac{e^{\Phi}\mathcal{G}(\rho, w_6)}{\sqrt{1+\partial_\rho w_6^2}}(\partial_\rho w_6)\right] - \sqrt{1+\partial_\rho w_6^2}\frac{d}{dw_6}\left[e^{\Phi}\mathcal{G}(\rho, w_6)\right] = 0\,, \tag{3.7}$$

where

$$\mathcal{G}(\rho, w_6) = \rho^3 \frac{((\rho^2 + w_6^2)^2 + b^4)((\rho^2 + w_6^2)^2 - b^4)}{(\rho^2 + w_6^2)^4}\,. \tag{3.8}$$

The final terms in the equation of motion is a “potential” like term that is evaluated to be

$$\frac{d}{dw_6}\left[e^{\Phi}\mathcal{G}(\rho, w_6)\right] = \frac{4\rho^3 w_6 b^4}{(\rho^2 + w_6^2)^5}\left(\frac{((\rho^2 + w_6^2)^2 + b^4)}{((\rho^2 + w_6^2)^2 - b^4)}\right)^{\Delta/2}(2b^4 - \Delta(\rho^2 + w_6^2)^2)\,. \tag{3.9}$$

Asymptotically at large radius these equations are just those in AdS with solution

$$w_6 = m + \frac{c}{\rho^2} + \cdots\,, \tag{3.10}$$

where m corresponds to the quark mass and c the condensate.

Numerical solutions for the regular embedding solutions are shown in fig. 3 as well as a plot of the parameter c vs m. We take $b = 1$ for these plots. The $m = 0$ solution breaks the symmetry in the $w_5 - w_6$ plane which is present asymptotically at large w where the D7 lies at the origin of the space - this is the geometric breaking of $\mathrm{U}(1)_A$.

Let us compare this case to that of the previous section where we showed that in the $\mathcal{N} = 4$ theory with scalar vev there was no chiral symmetry breaking. Here there is a non-zero value of c as m goes to zero. There is also a radial gap between the $m = 0$ embedding and the singularity. Whatever coordinate transformation we might make on the geometry this gap will remain and the U(1) symmetry in the $w_5 - w_6$ plane will be broken - this solution therefore definitely breaks chiral symmetry.

3.1 Spherical D7 Embedding

The above solutions in fig. 3 are calculated numerically, but it would be nice to be able to extract the chiral symmetry breaking behaviour and some dynamical information analytically. To do this we will look for minimum action solutions in the form of a circle in the $\rho - w_6$ plane plotted in fig. 3. The chiral symmetry breaking solutions naively look to match on to such a solution close to the singularity. Whether such an embedding falls into the singularity or is stabilized away from it will test whether the core of the geometry is repulsive to D7s and hence will be our test for chiral symmetry breaking. The distance it rests from the singularity will provide an estimate of the radial gap of the full embedding above. This gap corresponds to the dynamically generated quark mass at zero energy or the mass gap of the theory.

Concretely, we write the metric in the six w directions as

$$\sum_{i=1}^{6} dw_i^2 = dr^2 + r^2(d\alpha^2 + \cos^2\alpha d\phi^2 + \sin^2\alpha d\Omega_3^2)\,, \tag{3.11}$$

and embed the D7 brane on the Ω_3 and in α at constant $r = r_0$. The action of the D7 brane is

$$S_{\text{wrapped D7}} = -T_7 \int d^8\xi\; r^4 e^{\Phi(r)} g_{xx}^2(r) g_{ww}^2(r) \sqrt{1 + \frac{1}{r^2}\left(\frac{dr}{d\alpha}\right)^2}\,. \tag{3.12}$$

Note that in pure AdS or a multi-centre solution $g_{xx}^2 g_{ww}^2 = 1$, the dilaton is a constant, $dr/d\alpha$ is zero if r is fixed to some r_0 and so the action is simply r_0^4. In the supersymmetric case the circle collapses to the origin ($r_0 \to 0$) so here the core of the geometry is not repulsive which we take as evidence of the lack of chiral symmetry breaking.

In the Constable Myers geometry though we have

$$S_{\text{wrapped D7}} = -T_7 \int d^8\xi r^4 \left(\frac{r^4+b^4}{r^4-b^4}\right)^{1+\Delta/2} \left(\frac{r^4-b^4}{r^4}\right)^2 \sqrt{1 + \frac{1}{r^2}\left(\frac{dr}{d\alpha}\right)^2}\,. \tag{3.13}$$

Taking the equation of motion for $r(\alpha)$ we see that there is indeed a solution with $\frac{dr}{d\alpha} = 0$. The action is then minimized by the constant value of $r = r_0$ which is the root of

$$r_0^8 - b^4 \Delta r_0^4 + b^8 = 0 \tag{3.14}$$

which is real and greater than b. For $b = 1$ there is a minimum of this action away from the singularity showing that the singularity is repulsive to such a configuration. For $b = 1$ this gives $r_0 = 1.29$. In figure 4 we plot this solution and the massless quark solutions above. Comparison shows that the circular embedding provides a good approximation to the gap value for the D7 brane solutions we are interested in above. Generically how good this match is will depend on the form of the repulsive potential induced by the geometry. In this case the potential sets in steeply and the two solutions do match well.

It's clear that the full embedding comes close to matching to the spherical case in the infra-red. In fact, there is a small gap between these solutions. In [4] solutions were identified that lay closer to the singularity; these correspond to a second, higher action solution for each asymptotic value of the mass up to some critical mass. These solutions were identified with the chiral vacuum with $-\langle\bar{\psi}\psi\rangle$ whose energy is raised by the quark mass. If the quark mass were too big this vacuum is not even metastable and there is no

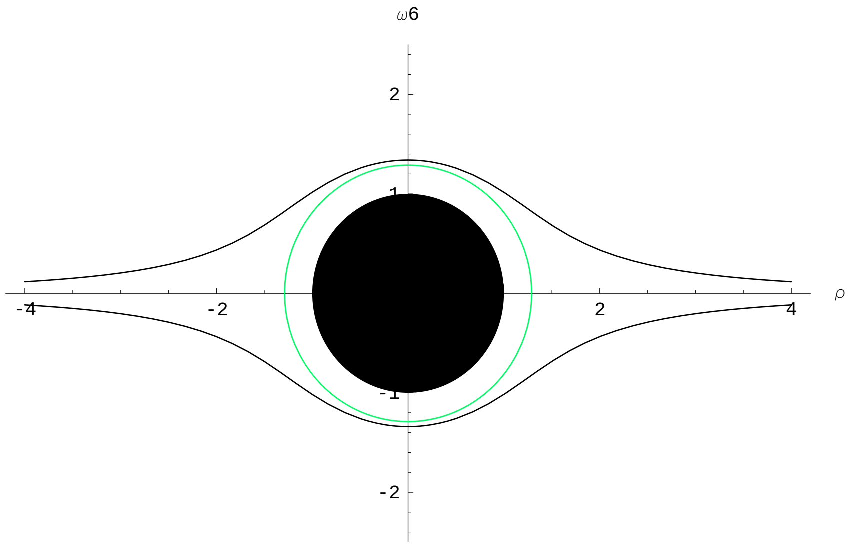

Figure 4: Plot of the minimum action spherical D7 embedding and the massless quark embeddings in the Constable Myers Geometry. The black circle represents the singularity in the geometry.

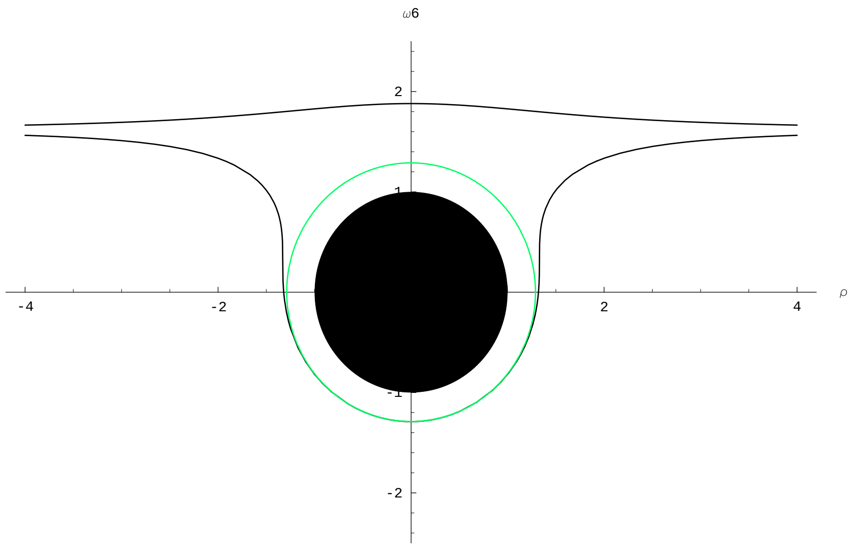

Figure 5: Plot of the minimum action spherical D7 embedding and a local minimum embedding action for a massive quark in the Constable Myers Geometry.

solution describing it. In figure 5 we plot this critical flow and the minimum action circular embedding showing that they overlap almost exactly. Overall though we conclude that the spherical embedding provides a good measure of the quark mass gap.

There is also some structure as b is changed. Writing w in powers of b removes b from everywhere in the metric except the expression for Δ in the dilaton. For Δ to be real requires

$$b \geq \left(\frac{1}{40}\right)^{1/8} R\,. \tag{3.15}$$

Now the quadratic equation (3.14) has a solution for the position of the minimum action, spherically embedded D7 which lies outside the radius b where the geometry is singular only if

$$b > \left(\frac{1}{24}\right)^{1/8} R\,. \tag{3.16}$$

There is therefore a small range of b that we can study where the spherical D7 falls into the singularity. For these values we expect the full D7 embedding corresponding to the quark fields to fall in too. Numerically this is what we find. In fact there is a few percent discrepancy between where a chiral symmetry breaking solution is lost and where the spherical embedded D7 enters the singularity corresponding to the slight mismatch seen above for the gap value in the exact method and the spherical embedding approximation. For these cases where the singularity is crossed we can come to no further understanding without more knowledge of the singularity.

4. A non-supersymmetric scalar deformation

Let us now turn to our first example of a non-supersymmetric gravity dual [41] that has not been previously studied in this context. The background we will look at is related to the $\mathcal{N}=4$ [31] embedding introduced in section 2. It was generated from a five dimensional supergravity flow which was lifted to ten dimensions in [41]. A five dimensional scalar field, λ, in the 20 representation of SO(6) is switched on and has the potential

$$V = -e^{-\frac{4\lambda(r)}{\sqrt{6}}} - 2e^{\frac{2\lambda(r)}{\sqrt{6}}}\,. \tag{4.1}$$

The scalar field λ acts as the source and vev of the field theory operator $\mathrm{Tr}(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2-2\phi_5^2-2\phi_6^2)$. Switching on a mass term will give rise to unbounded directions in the scalar potential and so, as with the Constable Myers case, this is not a realistic dual. Nevertheless it is an interesting case to see if the breaking of supersymmetry generates chiral symmetry breaking - we will see that in this case it does not.

The relevant equations of motion for the scalar field are given by the usual five dimensional field equations [35]:

$$\begin{aligned} \lambda''(r) + 4\lambda'(r)\sqrt{\frac{1}{6}(\lambda'(r)^2 - 2V)} &= \frac{\partial V}{\partial\lambda(r)}\,, \\ A'(r) &= \sqrt{\frac{1}{6}(\lambda'(r)^2 - 2V)}\,. \end{aligned} \tag{4.2}$$

The large r limit of this field is given by

$$\lambda = \mathcal{A} r e^{-2r} + \mathcal{B} e^{-2r}\,, \tag{4.3}$$

where $\mathcal{A}$ is interpreted as a source for our operator and $\mathcal{B}$ is the vev. The $\mathcal{N}=4$ deformation described earlier is the special case where only the vev is non-zero. Now, we are interested in the case where there is both a mass and condensate present. We plot numerical solutions of the five dimensional field equations in figure 6. Generically the flows either diverge in the

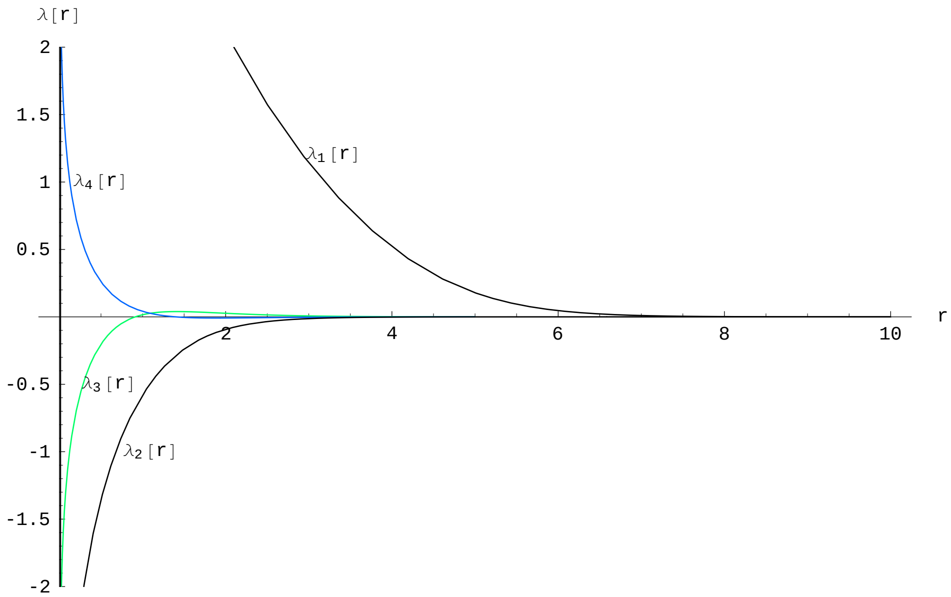

Figure 6: 4 different flows of the SUGRA field $\lambda(r)$ all of which become singular at r=0. λ_1 and λ_2 are the turning point flows between the flows of the form λ_3 and λ_4.

IR with $\lambda \to \pm\infty$. There is a boundary in the $\mathcal{A}$-$\mathcal{B}$ plane between these two behaviours. In figure 6 we show examples of each of these two behaviours λ_3, λ_4 and the unique flows that lie on the boundary between these two regimes. λ_1 describes this boundary in the positive $\mathcal{A}$ positive $\mathcal{B}$ quadrant whilst λ_2 describes the boundary in the quadrant where they are both negative. We will provide analytic forms for the IR of these flows below.

The ten dimensional lift of the full set of five dimensional solutions shares the form of the metric with the supersymmetric geometry so

$$ds^2 = \frac{\sqrt{X}}{\chi} e^{2A} dx_{//}^2 + \frac{\sqrt{X}}{\chi} \left(du^2 + \frac{R^2}{\chi^2} \left[d\theta^2 + \frac{\sin^2\theta}{X} d\phi^2 + \frac{\chi^6 \cos^2\theta}{X} d\Omega_3^2 \right] \right), \tag{4.4}$$

where the parameters are defined in exactly the same way as in the supersymmetric case (see eqn. (2.6)) and

$$\chi = e^{\lambda(r)/\sqrt{6}}. \tag{4.5}$$

Of course the solutions for χ and A differ from the supersymmetric case. The four form potential of the lift does not match that in the supersymmetric case but in neither case does it enter the DBI action of our D7 brane.

We can look for chiral symmetry breaking using both of the techniques we have seen in previous examples. The first method is more direct but a little less enlightening; we probe with a D7 brane by embedding in the $x_{//}, r, \Omega_3$ directions (the angle ϕ again provides the $\mathrm{U}(1)_A$ symmetry). The D7 action is given by eq. (2.10) and we will solve for flows in from the IR towards the UV with the symmetric boundary condition $v = const$ and calculate numerically the geometry of the brane. We saw in the supersymmetric example that we are not in the "correct" coordinate system to make the gauge theory living on the brane manifest and therefore to find the quark mass and condensate. In that case, we were

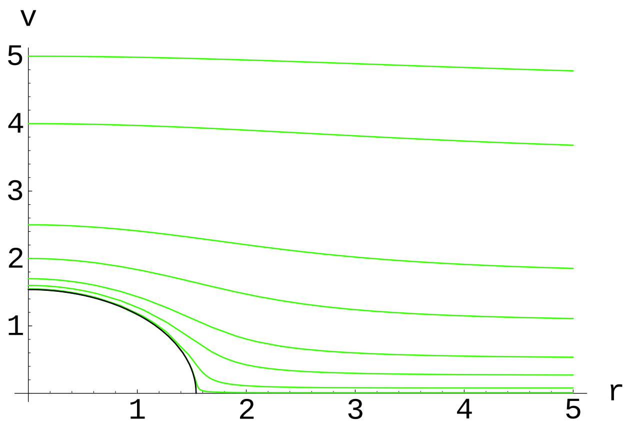

Figure 7: Sample solutions for a D7 brane embedding in the non-supersymmetric scalar deformation geometry showing the absence of a gap between the solutions and the singularity.

helped by the first order equations and could find the correct coordinate system. In this more general case, where χ and A are solutions of second order equations, we have no hint as to how to find the correct coordinates.

We can perform the embedding by calculating θ as a function of r, or we can change to the cartesian like set of coordinates, (v, r), given by the same change of variables as in the supersymmetric calculation eq. (2.11) and calculate the flow of $v(r)$. The action is then given in eq. (2.13). We solve for the D7 embedding with IR boundary condition $v(r) =$ constant. We plot some representative solutions in figure 7, which are in a background with positive $\mathcal{A}$ and $\mathcal{B}$ in eq. (4.3).

We find generically that for D7 embeddings that correspond to small quark masses in these geometries the flow wraps onto the surface of the singularity. As far as we can tell numerically there is a solution that hugs the v axis down to the singularity and then follows the singularity to $r = 0$. For this flow asymptotically both the mass and condensate are zero. We conclude that the geometry does not break chiral symmetries. Note that, as with the supersymmetric flows, naively a flow that hugs the singularity breaks the U(1) symmetry due to ϕ translations. However, it seems likely that there is a coordinate transformation that presses the singularity to the $v = 0$ axis as in the supersymmetric case. The absence of a gap between the D7 brane embedding and the singularity, which could not be removed by such a transformation, implies no clear signal of chiral symmetry breaking.

The numerical studies above are somewhat messy and it would be nice to have an analytic understanding of the results. Such results are provided by our spherical embedding method developed in section 3.1. We study the action of a circular D7 brane wrapped on $x_{//}, \Omega_3, \theta$ at constant r as it gets nearer to the singular region. We expect the full solutions above to match onto these solutions in the IR. If the spherical embedding collapses into the singularity then there will be D7 embeddings that flow in from infinite radius that also touch the singularity. This can be calculated from the numerical flows of the scalar field λ, but it is more satisfactory to be able to get an analytic form for the potential. It turns out that we can do this for the IR limit of the scalar flow equations. We find four different solutions for the scalar fields χ and A. These numbered solutions to χ are related to the four types of behaviour in figure 6 by $\chi = e^{\frac{\lambda}{\sqrt{6}}}$:

$$\begin{aligned}
\chi_1(r) &= \sqrt{\frac{45}{2}}\frac{1}{(r-r_s)}, & A_1(r) &= 4\log(c(r-r_s))\,, \\
\chi_2(r) &= \left(\frac{2}{3}(r-r_s)\right)^{1/2}, & A_2(r) &= \log(c(r-r_s))\,, \\
\chi_3(r) &= (b(r-r_s))^{1/4}\,, & A_3(r) &= \frac{1}{4}\log(c(r-r_s))\,, \\
\chi_4(r) &= (b(r-r_s))^{-1/4}\,, & A_4(r) &= \frac{1}{4}\log(c(r-r_s))\,,
\end{aligned} \tag{4.6}$$

where r_s is the singular radius and b and c are free parameters[1] We have checked that these are all of the possible solutions by numerically solving the flow equations with these as the IR conditions. We flow out to the UV with these boundary conditions and calculate the mass and condensate in this limit. As best we can tell these are indeed all the IR solutions of the SUGRA field equations.

Having found the analytic IR solutions, we can look at the action for the circular brane wrapping. We calculate the pullback for a brane with r and ϕ as the perpendicular directions with $r(\theta)$ and ϕ constant which gives:

$$\begin{aligned}
S_{\mathrm{DBI}} &= -T_7\int d^8\xi R^4\sqrt{\cos^2(\theta)+\chi(r(\theta))^6\sin^2(\theta)e^{4A(r(\theta))}}\; \times \\
&\quad \times \chi(r(\theta))\cos^3(\theta)\sqrt{1+\left(\frac{dr(\theta)}{d\theta}\right)^2\frac{\chi(r(\theta))^2}{R^2}}\,.
\end{aligned} \tag{4.7}$$

There are solutions where $r(\theta) = r_0$ and using the above IR solutions we can write down the action for each of these which gives

$$\begin{aligned}
S_{\mathrm{DBI1}} &\sim -\int d^8\xi (r_0-r_s)^{12}|\sin(\theta)||\cos^3(\theta)|\,, \\
S_{\mathrm{DBI2}} &\sim -\int d^8\xi (r_0-r_s)^{9/2}\cos^4(\theta)\,, \\
S_{\mathrm{DBI3}} &\sim -\int d^8\xi (r_0-r_s)^{5/4}\cos^4(\theta)\,, \\
S_{\mathrm{DBI4}} &\sim -\int d^8\xi |\sin(\theta)||\cos^3(\theta)|\,.
\end{aligned} \tag{4.8}$$

We can see that for the first three there is obviously a minimizing solution for which $r_0 = r_s$ meaning that the circular brane will fall into the singularity. This strongly suggests from

[1] We have found this asymptotic behaviour by looking for solutions of the form $\lambda(r) = a\log(b(r-r_s))$ as $r \to r_s$. In fact, the last two solutions are valid for any potential V which asymptotically smaller than $\frac{1}{(r-r_s)^2}$ as $r \to r_s$. In this case the supergravity equation reduces to the first order equation:

$$\lambda''(r) + 4\lambda'(r)\sqrt{\frac{1}{6}(\lambda'(r)^2)} \sim 0\,.$$

our earlier studies that there will not be any chiral symmetry breaking induced by this geometry. Indeed we saw above that numerical studies of the D7 embedding appropriate for the addition of quarks also suggest there is not chiral symmetry breaking here. It is not immediately apparent that the brane will collapse in the fourth solution, but we note that this solution interpolates from the UV for which the potential goes like e^{4r}. By performing this calculation numerically we find a monotonically decreasing potential indicating that the flat behaviour seen in the equation interpolates smoothly into the exponential behaviour meaning that the brane collapses with this solution as well.

We conclude that this non-supersymmetric geometry does not induce chiral symmetry breaking. The unrealistic nature of the geometry, having as it does an unbounded scalar potential, does not make this result overly concerning but it is interesting that supersymmetry breaking and chiral symmetry breaking do not appear to be necessarily directly linked.

5. $N = 2^*$ geometry

We now turn to a more complicated supersymmetric geometry describing the $\mathcal{N} = 2^*$ gauge theory [42, 43]. This theory is the $\mathcal{N} = 4$ theory with mass terms for two adjoint chiral matter fields. It has the massless fields of $\mathcal{N} = 2$ super Yang Mills and thus a two dimensional scalar moduli space. The supergravity description has a field corresponding to the fermionic mass and another which describes both the scalar masses and vev. The lift of the original 5d background to 10d has been made [42] and the solution in the Einstein frame is given by

$$ds^2 = \Omega^2 \left(e^{2A} dx_{//}^2 + dr^2 \right) + \frac{R^2 \Omega^2}{\chi^2} \left(\frac{d\theta^2}{c} + \chi^6 \cos^2\theta \left(\frac{\sigma_1^2}{cX_2} + \frac{\sigma_2^2 + \sigma_3^2}{X_1} \right) + \frac{\sin^2\theta}{X_2} d\phi^2 \right), \tag{5.1}$$

where

$$\begin{aligned} \Omega^2 &= \frac{(cX_1X_2)^{1/4}}{\chi}, \\ c &= \cosh 2\zeta, \\ X_1 &= \cos^2\theta + \chi^6 c \sin^2\theta, \\ X_2 &= c\cos^2\theta + \chi^6 \sin^2\theta, \\ C_{(4)} &= \frac{e^{4A} X_1}{4 g_s \chi^2} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3, \end{aligned} \tag{5.2}$$

and the axion/dilaton is given by:

$$\lambda = i\left(\frac{1-B}{1+B}\right) = C_{(0)} + ie^{-\Phi}, \qquad B = e^{2i\phi}\left(\frac{b^{1/4} - b^{-1/4}}{b^{1/4} + b^{-1/4}}\right), \qquad b = \frac{X_1}{X_2}\cosh 2\zeta. \tag{5.3}$$

In addition, there is the anti-symmetric two form, whose NSNS and RR parts are given respectively by

$$\begin{aligned} B_{(2)} &= a_3 \cos\phi\, \sigma_1 \wedge \mathrm{d}\phi + a_1 \sin\phi\, \mathrm{d}\theta \wedge \sigma_1 - a_2 \sin\phi\, \sigma_2 \wedge \sigma_3, \\ C_{(2)} &= -a_1 \cos\phi\, \mathrm{d}\theta \wedge \sigma_1 + a_2 \cos\phi\, \sigma_2 \wedge \sigma_3 + a_3 \sin\phi\, \sigma_1 \wedge \mathrm{d}\phi, \end{aligned} \tag{5.4}$$

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where

$$\begin{aligned}
a_1 &= R^2 \tanh 2\zeta \cos\theta \,, \\
a_2 &= R^2 \frac{\chi^6 \sinh 2\zeta}{X_1} \sin\theta \cos^2\theta \,, \\
a_3 &= R^2 \frac{\sinh 2\zeta}{X_2} \sin\theta \cos^2\theta \,.
\end{aligned} \tag{5.5}$$

The SUGRA fields ζ, A and $\chi = e^\alpha$ satisfy the equations of motion

$$\begin{aligned}
\frac{d\alpha}{dr} &= \frac{1}{3R}\left(\frac{1}{\chi^2} - \chi^4 \cosh(2\zeta)\right) , \\
\frac{dA}{dr} &= \frac{2}{3R}\left(\frac{1}{\chi^2} + \frac{1}{2}\chi^4 \cosh(2\zeta)\right) , \\
\frac{d\zeta}{dr} &= -\frac{1}{2R}\chi^4 \sinh(2\zeta) \,.
\end{aligned} \tag{5.6}$$

These have partial solutions:

$$\begin{aligned}
e^A &= k \frac{\chi^2}{\sinh(2\zeta)} \,, \\
\chi^6 &= \cosh(\zeta) + \sinh^2(2\zeta)\,(\gamma + \log(\tanh\zeta)) \,,
\end{aligned} \tag{5.7}$$

The solutions with large γ deform into $\mathcal{N} = 4$ solutions of the form seen in section 2 - here the scalar vev is so much larger than the supersymmetry breaking mass that the theory is effectively the $\mathcal{N} = 4$ theory. The smallest possible vev in the theory corresponds to the background with $\gamma = 0$ and we will concentrate on this case since it is the vacuum most distinct from the $\mathcal{N} = 4$ theory. Probing the geometry with a D3 brane [44, 45] shows that the $\theta = \pi/2$ plane corresponds to the moduli space. Further in [45] a set of coordinates were found on that moduli space that correspond to the physical coordinates for computing the β-function of the field theory. In these coordinates the singularity is again transformed to a branch cut.

As we have done in previous cases, we can analytically study the asymptotic solutions to the flow equations in the IR limit. The results are[2]

$$\begin{aligned}
\zeta(r) &= -\frac{3}{2}\log\left(\left(\frac{2}{3^5}\right)^{1/3} \frac{(r-r_s)}{R}\right) , \\
\chi(r) &= \frac{\sqrt{2}}{3}\sqrt{\frac{(r-r_s)}{R}} \,, \\
A(r) &= 4\log\frac{(r-r_s)}{R} + b \,.
\end{aligned} \tag{5.8}$$

[2]We find these by looking for real solutions to the flow equations which behave asymptotically like $\zeta(r) \sim a\log(b(r-r_s)/R), \chi(r) \sim c((r-r_s)/R)^d$ as $r \to r_s$. It can be shown that these solutions correspond to $\gamma = 0$ as, for $\gamma = 0$, we know from (5.7) that, for large ζ, $\chi \sim \left(\frac{4}{3}\right)^{\frac{1}{6}} e^{-\frac{\zeta}{3}}$, as can be checked for our solution.

These are a one parameter family of solutions as expected due to the fact that the flow equations in this case are first order. The parameter b is related to k by $b = \log k + \log(\frac{8}{2187})$. This is not a free parameter, but is fixed by requiring that this flows to a solution which is asymptotically $A(r) \to r$ in the UV.

Now consider including quark fields via a D7 brane probe in this geometry. The quark superfields have a superpotential coupling to the $\mathcal{N}=4$ adjoint scalars of the form $\tilde{Q}AQ$, where the adjoint field A is represented in the geometry by the two transverse directions to the D7 brane. Therefore in this geometry where the $\mathcal{N}=4$ fields have already been broken into $\mathcal{N}=2$ multiplets we must be careful to embed in such a way that we do not further break supersymmetry. The probe must lie perpendicular to the $\theta = \pi/2$ plane since that plane corresponds to the massless scalar fields. Were the probe at some angle to that plane the superpotential term would be with fields in a mix of superfields in terms of the breaking intrinsic in the geometry. In this configuration though we expect that we should be able to include arbitrary mass quarks and maintain supersymmetry.

If we were attempting to find the flow of the brane in from the UV, we would want to embed it in the $x_{//}, r, \alpha, \beta, \psi$ directions and the DBI action of this embedding can be calculated easily. However, it can be seen that the two perpendicular directions θ and ϕ will be dependent on both r and ψ, which will make the equations of motion for the brane computationally very difficult to solve. Instead, we will fall back on our wrapping technique. We write the DBI action of the brane in terms of an embedding where the perpendicular directions r and ϕ are functions of θ and ψ and we take a constant value of ϕ. This time by looking at the symmetries of the metric we know that the minimum action solution will not be spherical, however we can find out if there is a repulsive potential on the brane stopping it falling in on the singularity.

To calculate the potential of a D7 brane in this background we first rewrite the 3 differentials $\sigma_{1,2,3}$ in terms of the spherical coordinates α, β, ψ using $\sigma_1 = \frac{1}{2}(d\alpha + \cos\psi d\beta), \sigma_2 = \frac{1}{2}(\cos\alpha d\psi + \sin\alpha\sin\psi d\beta), \sigma_3 = \frac{1}{2}(-\sin\alpha d\psi + \cos\alpha\sin\psi d\beta)$. This gives a metric of the form

$$\begin{aligned} ds^2 =& \ \Omega^2(e^{2A}dx_{//}^2 + dr^2) + \frac{R^2\Omega^2}{\chi^2} \times \\ &\times \left(\frac{1}{c}d\theta^2 + \frac{\sin^2\theta}{X_2}d\phi^2 + \frac{\chi^6\cos^2\theta}{4} \times \right. \\ &\left. \times \left(\frac{d\psi^2}{X_1} + \frac{d\alpha^2}{cX_2} + d\beta^2\left(\frac{\sin^2\psi}{X_1} + \frac{\cos^2\psi}{cX_2}\right) + \frac{2d\alpha d\beta\cos\psi}{cX_2} \right)\right). \end{aligned} \tag{5.9}$$

In fact even this configuration is hard to study because of the forms present in the geometry. For simplicity we will place the D7 probe at $\phi = n\pi$ where the axion vanishes, and the dilation is given by

$$e^{\Phi} = \sqrt{\frac{cX_1}{X_2}}. \tag{5.10}$$

In addition, the NSNS two form is zero.

The DBI action for this configuration is given by

$$\begin{aligned} S_{DBI} &= -T_7 \int d^8\xi \frac{R^4}{4} e^{4A} c X_1 \cos^2\theta |\sin\psi| \times \\ &\quad \times \sqrt{\left(\frac{\chi^2\cos^2\theta}{4cX_1} + \frac{\chi^4\cos^2\theta}{4R^2X_1}\left(\frac{\partial r}{\partial\theta}\right)^2 + \frac{1}{cR^2\chi^2}\left(\frac{\partial r}{\partial\psi}\right)^2 \right)}. \end{aligned} \tag{5.11}$$

We also have to consider the Wess-Zumino part of the action for the D7 brane. As there are no gauge fields living on the brane, this is given by

$$S_{WZ} = \mu_7 \int_{\mathcal{M}_8} (C_{(8)} + C_{(6)} \wedge B_{(2)}) . \tag{5.12}$$

For $\phi = n\pi$ this will be zero as the dual of $C_{(8)}$ (the axion) is zero and $B_{(2)}$ is also zero.

As before, we are only interested in the IR behaviour of the potential felt by the brane so we can use our analytic solution in the above equation. Ignoring proportionality factors, we find that for a constant r_0 solution

$$V_{IR} \sim (r_0 - r_s)^{15} |\cos^3\theta| |\sin\psi| \sqrt{5 + \cos 2\theta} . \tag{5.13}$$

We can see clearly that, as expected, this supersymmetric background, when probed in this particular way, does not have the signature of chiral symmetry breaking. There is no potential stopping the spherical D7 brane falling onto the singularity and the situation is analagous to that we found in the $\mathcal{N} = 4$ geometries.

6. The Yang Mills* geometry

Finally we will turn our spherical D7 embedding technique on a geometry where physically one might expect chiral symmetry breaking. The YM^* geometry [46] was developed as a model of non-supersymmetric Yang Mills theory. The UV of the theory is the $\mathcal{N} = 4$ gauge theory but then at a scale $\mathcal{M}$ a mass term is introduced for the four adjoint fermions, λ_i. It is also possible to include in the solutions a vev for the operator $\sum_i \lambda_i \lambda_i$. One would expect some dynamical determination of this condensate in terms of $\mathcal{M}$ but the supergravity solution does not clearly provide this link - we will investigate this whole space of geometries therefore.

The YM^* geometry was originally constructed as a 5d supergravity solution but then lifted to a full 10d solution. In 10d D3 brane probe analysis indicates that the six adjoint scalars of the $\mathcal{N} = 4$ theory acquire masses radiatively as one would expect since supersymmetry is broken. In [47] the glueball spectrum and string tension properties were analyzed. For the geometries with a fermion mass and small or vanishing condensate, a discrete glueball spectrum was found though probe strings fell onto the singularity. The interior of the geometry is therefore still ill understood, and possibly is also incomplete since the restricted 5d solution on which the geometry is built may not have sufficient freedom to describe the full non-supersymmetric theory. Nevertheless, this is a prime candidate geometry to examine for chiral symmetry breaking since it describes a model that shows some very QCD like qualities.

In the SUGRA theory the fermion mass terms correspond to turning on a scalar in the 10 of SO(6). As in the other examples, we can solve the 5d SUGRA equations numerically using the relevant field equations.

$$
\begin{aligned}
\lambda''(r) + 4A'(r)\lambda'(r) &= \frac{\partial V}{\partial \lambda}\,, \\
A'(r) &= \sqrt{\frac{1}{6}(\lambda'(r)^2 - 2V)}\,,
\end{aligned}
\tag{6.1}
$$

with

$$
V = -\frac{3}{2}(1 + \cosh(\lambda(r))^2)\,. \tag{6.2}
$$

The AdS limit of this field has solutions

$$
\lambda(r) = \mathcal{M}e^{-r} + \mathcal{C}e^{-3r}\,, \tag{6.3}
$$

with $\mathcal{M}$ and $\mathcal{C}$, the mass and condensate of the operator $\sum_i \lambda_i\lambda_i$ respectively. We will be particularly interested in the large negative r limit of the space corresponding to the IR of the gauge theory. We can try to find analytic solutions in this limit as we did in the previous section. The solutions we have found are

$$
\begin{aligned}
\lambda_{\mathrm{IR},1}(r) &= -\log\left(\sqrt{\frac{9}{20}}(r - r_s)\right), & A(r) &= \frac{2}{3}\log(r - r_s)\,, \\
\lambda_{\mathrm{IR},2}(r) &= \log\left(\sqrt{\frac{9}{20}}(r - r_s)\right), & A(r) &= \frac{2}{3}\log(r - r_s)\,, \\
\lambda_{\mathrm{IR},3}(r) &= -\frac{\sqrt{6}}{4}\log(a(r - r_s))\,, & A(r) &= \frac{1}{4}\log(r - r_s)\,, \\
\lambda_{\mathrm{IR},4}(r) &= +\frac{\sqrt{6}}{4}\log(a(r - r_s))\,, & A(r) &= \frac{1}{4}\log(r - r_s)\,,
\end{aligned}
\tag{6.4}
$$

where r_s is the radius where the flow becomes singular in the IR. In the $\mathcal{C}$ vs $\mathcal{M}$ plane there are regions where the λ flows diverge positively and negatively described by the second two solutions. There is then a unique flow on the boundary which in the positive quadrant is described by λ_1 and in the negative quadrant by λ_2. Numerically the unique flow lies at least close to the boundary condition $\mathcal{C} = 0$.

The 10d lift of the YM^* background is given by

$$
\begin{aligned}
ds^2_{10} &= \xi^{1/2}(e^{2A}dx^2 + dr^2) + \xi^{-3/2}(\xi^2 d\alpha^2 + \sin^2\alpha F_+ d\Omega_+^2 + \cos^2\alpha F_- d\Omega_-^2)\,, \\
d\Omega_\pm^2 &= d\theta_\pm^2 + \sin^2\theta_\pm d\phi_\pm^2\,, \\
\xi^2 &= \cosh^4\lambda(r) + \sinh^4\lambda(r)\cos^2 2\alpha\,, \\
F_\pm &= \cosh^2\lambda(r) \pm \sinh^2\lambda(r)\cos 2\alpha\,, \\
A\pm &= \frac{\sinh 2\lambda(r)}{\cosh^2\lambda(r) \pm \cos 2\alpha \sinh^2\lambda(r)}\,,
\end{aligned}
$$

$$
\begin{aligned}
B &= \frac{\sinh^2 \lambda(r) \cos 2\alpha}{\cosh^2 \lambda(r) + \xi}\,, \\
e^{-\Phi} &= \frac{1-B}{1+B}\,.
\end{aligned}
\tag{6.5}
$$

Now consider including quark fields via a D7 brane probe in this geometry. The probe would naturally lie in the $x_{//}$ and r directions and then wrap a three sphere in the deformed five sphere. For example we could wrap one of the two spheres (e.g. Ω_-) and fill the angle α leaving us to find the embedding θ_+. The angle ϕ_+ provides the $\mathrm{U}(1)_A$ symmetry of the quarks. Clearly θ_+ will be a function of both r and α on the D7 world volume. In this complicated geometry it is too difficult to find the full embedding.[3] It is more straightforward though to embed a D7 spherically on $x_{//}, \Omega_-, \alpha$ and θ_+ at fixed ϕ_+. We treat the radius of this sphere r as our embedding coordinate. The DBI action for this configuration is

$$
S_{DBI} = -T_{D7} \int d^8\xi\; e^{\Phi} \sqrt{-M} \left((\partial_{\theta_+} r)^2 + \left(1 + (\partial_\alpha r)^2\right) \frac{\sin^2 \alpha}{F_-} \right)^{\frac{1}{2}}\,, \tag{6.8}
$$

where

$$
\sqrt{-M} = e^{4A} \sqrt{\cos^4 \alpha F_-^2 \sin^2 \theta_- + \frac{A_-^2}{4} \sin^6 \alpha \cos^2 \theta_- \xi^3}\,. \tag{6.9}
$$

The Wess-Zumino contribution eq. (5.12) of the action must also be studied. However, the integral over $C_{(8)}$ is zero as it's dual, the axion, is zero and the integral over $C_{(6)} \wedge B_{(2)}$ will also be zero since $C_{(6)}$ is dual to $C_{(2)}$ ($\mathrm{d}C_{(6)} = *\mathrm{d}C_{(2)}$) and $C_{(2)}$ has a basis $\mathrm{d}\theta_+ \wedge \mathrm{d}\phi_+$, whereas $B_{(2)}$ has a basis $\mathrm{d}\theta_- \wedge \mathrm{d}\phi_-$. This means that we will end up with a wedge product of identical basis one forms which will be zero. For this embedding we can therefore drop these terms.

The symmetries of our action imply that there are solutions where r is just a function of α which is to be compared with the more complicated full embedding where $\theta_+(r, \alpha)$. Numerical analysis will therefore be simpler in this case.

As our variable is not dependent on θ_-, we must integrate over this quantity in the action before we can try to find a solution. This integral gives

$$
\begin{aligned}
S_{\mathrm{DBI}} = -T_7 \int d^7\xi\; e^{\Phi} e^{4A} 2F_- \cos^2 \alpha\; \times \\
\times \mathrm{Elliptic}E \left(1 - \frac{A_-^2 \sin^6 \alpha \zeta^3}{4F_-^2 \cos^4 \alpha} \right) \sqrt{(\partial_{\theta_+} r)^2 + (1 + (\partial_\alpha r)^2) \frac{\sin^2 \alpha}{F_-}}\,.
\end{aligned}
\tag{6.10}
$$

[3] Note that the usual AdS geometry can be written in the same coordinate system as YM^* so the metric is

$$
ds^2 = e^{2r} dx_{//}^2 + dr^2 + d\alpha^2 + \cos \alpha d\Omega_+^2 + \sin \alpha d\Omega_-^2\,. \tag{6.6}
$$

The flat D7 embedding of section 2 in these coordinates is then given by

$$
\theta_- = \arcsin \frac{m e^{-r}}{\sin \alpha} \tag{6.7}
$$

which is itself a complicated function. Even in this case sophisticated numerical techniques would be needed to find this solution.

The simplest analysis we can perform is just to look at the potential felt by a brane of fixed radius r_0. This will not be a solution of the equations of motion but will show us whether there is a repulsion in the geometry to such a configuration collapsing. For our four analytic solutions, we find two sorts of behaviour in the IR. These are given by the potentials

$$V_{\text{circle}} = T_7 \int d^7\xi \ (r_0 - r_s)^{\frac{2}{3}} \ EllipticE\left(1 - \frac{18}{5}(r_0 - r_s)^2 |\cot\alpha|\right) \sqrt{|\cos^5\alpha \sin^3\alpha|}\,, \quad (6.11)$$

corresponding to $\lambda_{IR,1}$ and $\lambda_{IR,2}$ and

$$\begin{aligned} V_{circle} &= T_7 \int d^7\xi \ (b(r_0 - r_s))^{1-\sqrt{3/2}} \times \\ &\quad \times \text{Elliptic}E\left(1 - 8(b(r_0 - r_s))^{\sqrt{3/2}} |\cot\alpha|\right) \sqrt{|\cos^5\alpha \sin^3\alpha|}\,, \end{aligned} \quad (6.12)$$

corresponding to $\lambda_{IR,3}$ and $\lambda_{IR,4}$. For both of these solutions the elliptic integral tends to a constant in the IR so we can see explicitly the behaviour of the potential as a function of $r_0 - r_s$. In the first case, corresponding to a line of solutions in the $\mathcal{M}$ vs $\mathcal{C}$ plane the embedding collapses onto the singularity. For the remainder of the $\mathcal{M}$ vs $\mathcal{C}$ space the probe is repelled from the singularity in the IR. In the UV where the geometry returns to AdS the potential always pushes the field into the IR so there must be a stable configuration away from the singularity. This suggests that the majority of parameter space in the model will give rise to chiral symmetry breaking.

We know however from the α dependence of the action that the solution of the wrapped brane will not have a circular symmetry. Therefore, although we know that there is a repulsive potential at least somewhere around the singularity, we don't yet know what form the brane embedding will take. To calculate this, we have used a numerical relaxation method. We discretise the points on the brane parameterised by the angle α and write the action for $r(\alpha)$ in the discretised form with a starting guess for the solution. We then minimise the action with respect to all points on the lattice, giving us N coupled equations for N lattice points. Because we are looking at a wrapping solution, we also have a boundary condition that the first and last points in α are at the same value of r. We performed this calculation using Mathematica. The resulting configuration of the brane is shown in figure 8 for a generic flow of the type we have seen with a repulsive potential. We see that the repulsive potential is present away from $\alpha = n\pi/2$ but vanishes at precisely $n\pi/2$ at which points the brane can fall in on the singularity. This is obviously an added complication which muddies the result. Consider beginning with a flat D7 brane far from the singularity which would include a heavy quark field in the theory. As the brane is brought in towards the singularity the repulsion centred at $\alpha = \pi/4$ will stop the brane moving to the origin of the space as sketched in figure 9 implying a chiral symmetry breaking set of UV boundary conditions. The outstanding question is whether the probe will touch the singularity at $\alpha = \pi/2$. We can not address this completely within our analysis. Of course touching the singularity may not be a disaster if it is simply indicating the presence of a fuzzy configuration of D3s - some brane bound state may form there. We

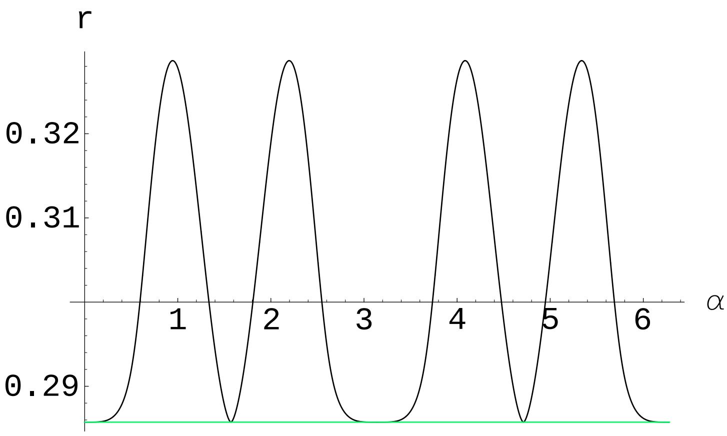

Figure 8: Sample result of a numerical solution for the determination of a spherical D7 brane embedding r vs α that wraps the YM^* singularity shown by the lower straight line.

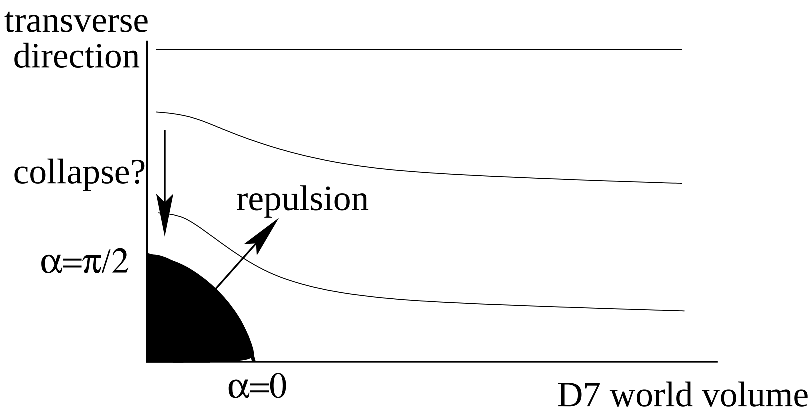

Figure 9: A sketch of how a flat D7 brane embedding is expected to behave in YM^* as the brane is brought close to the singularity. The repulsive potential away from $\alpha = n\pi/2$ will induce a chiral symmetry breaking like configuration although the D7 may collapse into the singularity at $\alpha = \pi/2$.

tentatively conclude that YM^* generically does break chiral symmetries although there are outstanding issues to be understood.

7. Summary

Quark fields can be introduced into the AdS/CFT Correspondence and its deformations via D7 brane probes. The mass and any chiral condensate induced can be read off from the asymptotic behaviour of the scalar in the D7 DBI action describing its embedding. We have shown that in fact this prescription must be tempered by the possibility of making coordinate transformations that alter the asymptotic behaviour of the field. In the FGPW geometry [31] that describes the $\mathcal{N}=4$ gauge theory on its moduli space we saw that the D7 embedding flows fill the whole space away from the central singularity of the geometry. A transformation that takes the singularity to a branch cut would then remove the signal for chiral symmetry breaking. In this case that is precisely the coordinate transformation that has been previously identified as necessary in the literature. In contrast, in the non-supersymmetric dilaton flow [37] in which chiral symmetry breaking has previously been studied [1], there are D7 embeddings for all possible quark masses that lie separated from the core singularity. Here no coordinate transformation can remove the symmetry breaking embedding.

To look at more complicated metrics where the full D7 embedding is numerically too involved to find, we developed a simple spherical D7 embedding in the dilaton driven flow. The full embedding appropriate for quark fields matches on to this spherical embedding in the IR. In the dilaton flow case, we could analytically find the potential for the spherical embedding and use it to show that the core of the geometry is repulsive. This repulsion drives the D7 brane into a chiral symmetry breaking configuration. We could also compute an analytic estimate of the quark mass gap.

We then used this spherical embedding technique to test three new geometries for chiral symmetry breaking behaviour. The first is a non-supersymmetric version of the FGPW background [41] in which an unbounded scalar mass is included in the gauge theory. D7 branes in this geometry behaved precisely as those in the supersymmetric version of the FGPW geometry leaving no gap between the flows and the singularity. The spherical embedding technique showed there was no potential stopping the spherical D7 falling onto the singularity. We conclude that this geometry does not break chiral symmetries.

We next looked at a spherical D7 embedding in the $\mathcal{N}=2^*$ geometry [42] and found again that the D7 falls in on the singularity. This implies that for a supersymmetric embedding of a D7 for the inclusion of quarks in the geometry there would be no chiral symmetry breaking as we would expect.

Finally we studied a spherical D7 embedding in the non-supersymmetric Yang Mills* geometry [46]. For most of the parameter space we found the central singularity of the geometry displayed a repulsive potential. This would imply chiral symmetry breaking embeddings for quark fields. There are special angles at which the repulsion vanishes but most probably this will not effect the conclusion.

Interestingly we have found chiral symmetry breaking in only those theories that are both non-supersymmetric and have a running dilaton. The former matches our expectations but the generality of the latter point is unclear. We believe that our spherical embedding technique will provide a simple test for chiral symmetry breaking in the more complicated geometries that will be needed to describe QCD realistically.

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References

[1] N.J. Evans and J.P. Shock, *Chiral dynamics from AdS space*, *Phys. Rev.* **D 70** (2004) 046002 [hep-th/0403279];
J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik and I. Kirsch, *Chiral symmetry breaking and pions in non-supersymmetric gauge/gravity duals*, *Phys. Rev.* **D 69** (2004) 066007 [hep-th/0306018].

[2] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Towards a holographic dual of large-N_c QCD*, *J. High Energy Phys.* **05** (2004) 041 [hep-th/0311270].

[3] K. Ghoroku and M. Yahiro, *Chiral symmetry breaking driven by dilaton*, *Phys. Lett.* **B 604** (2004) 235 [hep-th/0408040].

[4] N.J. Evans and J.P. Shock, *Chiral dynamics from AdS space*, *Phys. Rev.* **D 70** (2004) 046002 [hep-th/0403279].

[5] D. Bak and H.U. Yee, *Separation of spontaneous chiral symmetry breaking and confinement via AdS/CFT correspondence*, *Phys. Rev.* **D 71** (2005) 046003 [hep-th/0412170].

[6] A. Ilderton, *Radial evolution in anti-de Sitter spacetime*, hep-th/0501218.

[7] J.L.F. Barbon, C. Hoyos, D. Mateos and R.C. Myers, *The holographic life of the eta'*, *J. High Energy Phys.* **10** (2004) 029 [hep-th/0404260].

[8] A. Armoni, *Witten-Veneziano from Green-Schwarz*, *J. High Energy Phys.* **06** (2004) 019 [hep-th/0404248].

[9] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[10] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[11] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[12] A. Karch and E. Katz, *Adding flavor to AdS/CFT*, *J. High Energy Phys.* **06** (2002) 043 [hep-th/0205236].

[13] A. Karch, E. Katz and N. Weiner, *Hadron masses and screening from AdS Wilson loops*, *Phys. Rev. Lett.* **90** (2003) 091601 [hep-th/0211107].

[14] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Meson spectroscopy in AdS/CFT with flavour*, *J. High Energy Phys.* **07** (2003) 049 [hep-th/0304032].

[15] T. Sakai and J. Sonnenschein, *Probing flavored mesons of confining gauge theories by supergravity*, *J. High Energy Phys.* **09** (2003) 047 [hep-th/0305049].

[16] H. Nastase, *On Dp-Dp$\,+\,4$ systems, QCD dual and phenomenology*, hep-th/0305069.

[17] X.-J. Wang and S. Hu, *Intersecting branes and adding flavors to the maldacena- Núñez background*, *J. High Energy Phys.* **09** (2003) 017 [hep-th/0307218].

[18] P. Ouyang, *Holomorphic D7-branes and flavored $N = 1$ gauge theories*, *Nucl. Phys.* **B 699** (2004) 207 [hep-th/0311084].

[19] C. Núñez, A. Paredes and A.V. Ramallo, *Flavoring the gravity dual of $N = 1$ Yang-Mills with probes*, *J. High Energy Phys.* **12** (2003) 024 [hep-th/0311201].

[20] S. Hong, S. Yoon and M.J. Strassler, *Quarkonium from the fifth dimension*, *J. High Energy Phys.* **04** (2004) 046 [hep-th/0312071].

[21] B.A. Burrington, J.T. Liu, L.A. Pando Zayas and D. Vaman, *Holographic duals of flavored $N = 1$ super Yang-Mills: beyond the probe approximation*, *J. High Energy Phys.* **02** (2005) 022 [hep-th/0406207].

[22] J. Erdmenger and I. Kirsch, *Mesons in gauge/gravity dual with large number of fundamental fields*, *J. High Energy Phys.* **12** (2004) 025 [hep-th/0408113].

[23] D. Arean, D. Crooks and A.V. Ramallo, *Supersymmetric probes on the conifold*, *J. High Energy Phys.* **11** (2004) 035 [hep-th/0408210].

[24] M. Kruczenski, L.A.P. Zayas, J. Sonnenschein and D. Vaman, *Regge trajectories for mesons in the holographic dual of large-N_c QCD*, hep-th/0410035.

[25] S. Kuperstein, *Meson spectroscopy from holomorphic probes on the warped deformed conifold*, hep-th/0411097.

[26] Z. Guralnik, *Strong coupling dynamics of the Higgs branch: rolling a Higgs by collapsing an instanton*, hep-th/0412074.

[27] A. Paredes and P. Talavera, *Multiflavour excited mesons from the fifth dimension*, hep-th/0412260.

[28] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda and R. Marotta, *$N = 2$ gauge theories on systems of fractional D3/D7 branes*, *Nucl. Phys.* **B 621** (2002) 157 [hep-th/0107057].

[29] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda and R. Marotta, *More anomalies from fractional branes*, *Phys. Lett.* **B 540** (2002) 104 [hep-th/0202195].

[30] T. Sakai and S. Sugimoto, *Low energy hadron physics in holographic QCD*, hep-th/0412141.

[31] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Continuous distributions of D3-branes and gauged supergravity*, *J. High Energy Phys.* **07** (2000) 038 [hep-th/9906194].

[32] M. Günaydin, L.J. Romans and N.P. Warner, *Gauged $N = 8$ supergravity in five-dimensions*, *Phys. Lett.* **B 154** (1985) 268.

[33] M. Pernici, K. Pilch and P. van Nieuwenhuizen, *Gauged $N = 8$ $D = 5$ supergravity*, *Nucl. Phys.* **B 259** (1985) 460.

[34] M. Günaydin, L.J. Romans and N.P. Warner, *Compact and noncompact gauged supergravity theories in five-dimensions*, *Nucl. Phys.* **B 272** (1986) 598.

[35] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Confinement and condensates without fine tuning in supergravity duals of gauge theories*, *J. High Energy Phys.* **05** (1999) 026 [hep-th/9903026].

[36] J. Babington, N.J. Evans and J. Hockings, *Secrets of the metric in $N = 4$ and $N = 2^*$ geometries*, *J. High Energy Phys.* **07** (2001) 034 [hep-th/0105235].

[37] N.R. Constable and R.C. Myers, *Exotic scalar states in the AdS/CFT correspondence*, *J. High Energy Phys.* **11** (1999) 020 [hep-th/9905081].

[38] S.S. Gubser, *Dilaton-driven confinement*, hep-th/9902155.

[39] A. Kehagias and K. Sfetsos, *On asymptotic freedom and confinement from type-IIB supergravity*, *Phys. Lett.* **B 456** (1999) 22 [hep-th/9903109].

JHEP03(2005)005

[40] H. Liu and A.A. Tseytlin, *D3-brane d-instanton configuration and $N = 4$ super YM theory in constant self-dual background*, *Nucl. Phys.* **B 553** (1999) 231 [hep-th/9903091].

[41] J. Babington, D.E. Crooks and N.J. Evans, *A non-supersymmetric deformation of the AdS/CFT correspondence*, *J. High Energy Phys.* **02** (2003) 024 [hep-th/0207076].

[42] K. Pilch and N.P. Warner, *$N = 2$ supersymmetric RG flows and the iib dilaton*, *Nucl. Phys.* **B 594** (2001) 209 [hep-th/0004063].

[43] A. Brandhuber and K. Sfetsos, *An $N = 2$ gauge theory and its supergravity dual*, *Phys. Lett.* **B 488** (2000) 373 [hep-th/0004148].

[44] N.J. Evans, C.V. Johnson and M. Petrini, *The enhancon and $N = 2$ gauge theory/gravity RG flows*, *J. High Energy Phys.* **10** (2000) 022 [hep-th/0008081].

[45] A. Buchel, A.W. Peet and J. Polchinski, *Gauge dual and noncommutative extension of an $N = 2$ supergravity solution*, *Phys. Rev.* **D 63** (2001) 044009 [hep-th/0008076].

[46] J. Babington, D.E. Crooks and N.J. Evans, *A stable supergravity dual of non-supersymmetric glue*, *Phys. Rev.* **D 67** (2003) 066007 [hep-th/0210068].

[47] R. Apreda, D.E. Crooks, N.J. Evans and M. Petrini, *Confinement, glueballs and strings from deformed AdS*, *J. High Energy Phys.* **05** (2004) 065 [hep-th/0308006].